

Write and **prove** a formula for $\cosh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

SCORE: ____ / 6 PTS

$$\cosh x \cosh y - \sinh x \sinh y \quad (2)$$

$$= \left[\frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2} \right] \quad (1)$$

$$= \left[\frac{e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y} - (e^{x+y} - e^{x-y} - e^{y-x} + e^{-x-y})}{4} \right] \quad (2)$$

$$= \frac{2e^{x-y} + 2e^{y-x}}{4}$$

$$= \left[\frac{e^{x-y} + e^{-(x-y)}}{2} \right] \quad (1)$$

$$= \cosh(x-y)$$

Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$.

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$$\frac{e^x - e^{-x}}{2} = 1$$

$$\left[\frac{z - \frac{1}{z}}{2} = 1 \right] \textcircled{1}$$

$$z - \frac{1}{z} = 2$$

$$z^2 - 1 = 2z$$

$$\left[z^2 - 2z - 1 = 0 \right] \textcircled{1}$$

$$z = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$= \left[\frac{2 \pm \sqrt{8}}{2} \right] \textcircled{\frac{1}{2}}$$

EITHER

IS OK

$$= \left[\frac{2 \pm 2\sqrt{2}}{2} \right] = \left[1 \pm \sqrt{2} \right] \textcircled{1}$$

$$z = e^x > 0 \textcircled{1}$$

$$\text{SO } z = 1 + \sqrt{2}$$

$$e^x = 1 + \sqrt{2}$$

$$\left[x = \ln(1 + \sqrt{2}) \right] \textcircled{\frac{1}{2}}$$

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: ____ / 4 PTS

- [a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

- [b] Divide both sides of that identity by $\sinh^2 x$ and simplify.

$$\coth^2 x - 1 = \operatorname{csch}^2 x \quad (1)$$

- [c] If $\coth x = 3$, find $\sinh x$.

$\left(\frac{1}{2}\right)$
EITHER
IS OK

$$\begin{aligned} 3^2 - 1 &= \operatorname{csch}^2 x \\ \operatorname{csch}^2 x &= 8 \\ \left(\frac{1}{2}\right) \operatorname{csch} x &= \pm 2\sqrt{2} \end{aligned}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

SINCE $\coth x$ AND $\cosh x > 0$, $\left(\frac{1}{2}\right)$

THEREFORE $\sinh x > 0$

$$\text{SO } \operatorname{csch} x = 2\sqrt{2}$$

$$\left(\frac{1}{2}\right) \sinh x = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \quad \left(\frac{1}{2}\right)$$

Rewrite $\operatorname{sech}(3\ln 2)$ in terms of exponential functions and simplify.

SCORE: ____ / 3 PTS

$$\boxed{\frac{2}{e^{3\ln 2} + e^{-3\ln 2}}} = \frac{2}{e^{\ln 8} + e^{\ln \frac{1}{8}}} = \boxed{\frac{2}{8 + \frac{1}{8}}} = \boxed{\frac{16}{65}}$$

① ① ①

Prove that $g(x) = \ln(x + \sqrt{x^2 + 1})$ is the inverse of $f(x) = \sinh x$ by simplifying $g(f(x))$.

SCORE: ____ / 5 PTS

$$\begin{aligned} & \ln(\sinh x + \sqrt{\sinh^2 x + 1}) \quad (1) \\ &= \ln(\sinh x + \sqrt{\cosh^2 x}) \\ &= \ln(\sinh x + \cosh x) \quad (1) \\ &= \ln\left(\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}\right) \quad (1) \\ &= \ln \frac{2e^x}{2} \\ &= \ln e^x \quad (1) \\ &= x \quad (1) \end{aligned}$$